A Few Questions Concerning $\gamma$-Graphs

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Abstract

As introduced in the paper by Fricke, et. al., given a graph $G = (V, E)$, the $\gamma$-graph $G(\gamma) = (V(\gamma), E(\gamma))$ is the graph whose vertex set corresponds in a one-to-one way with the $\gamma$-sets, or minimum-cardinality dominating sets, of $G$. Two $\gamma$-sets, say $D_1$ and $D_2$, are adjacent in $E(\gamma)$ if there exists a vertex $v \in D_1$ and a vertex $w \in D_2$ such that $v$ is adjacent to $w$ and $D_1 = D_2 - \{w\} \cup \{v\}$, or equivalently, $D_2 = D_1 - \{v\} \cup \{w\}$. In this talk we investigate two open questions regarding these $\gamma$-graphs. The first concerns whether every graph $H$ is the $\gamma$-graph of some graph $G$. We show that for every graph $H$, there exists a graph $G$ such that $H \simeq G(\gamma)$. The second question concerns when $G(\gamma)$ is disconnected. Here we show that a graph, $G$, must have at least six vertices for $G(\gamma)$ to be disconnected, and we characterize the graphs on six vertices so that $G(\gamma)$ is disconnected.